

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS



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## COMPARISON OF VARIOUS METHODS FOR COMPUTING DRAG FROM WAKE SURVEYS

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ADVANCE RESTRICTED REPORT

COMPARISON OF VARIOUS METHODS FOR COMPUTING  
DRAG FROM WAKE SURVEYS

By Wallace F. Davis

SUMMARY

The various equations for computing profile drag by the momentum method are examined, and the errors arising from complete or partial neglect of compressibility effects in the Jones equation (R. & M. No. 1688) and the Bicknell equation (NACA Rep. No. 667) are evaluated. The integrating method of Silverstein and Katzoff (Jour., Aero. Sciences, vol. 7, no. 7, May 1940) is shown to be accurate over a wide range of Mach number and wake shapes.

INTRODUCTION

Equations for the computation of the profile drag of a body from an integration of the momentum defect in its wake were developed independently by A. Betz in 1925 (reference 1) and by B. M. Jones in 1936 (reference 2). Some of the assumptions made in the equations are not rigorously correct, but it has been shown that the errors involved are small (references 3 and 4). Both Jones and Betz neglected compressibility effects, and it was necessary to adapt their equations to compressible flow because of the high velocities encountered in modern wind-tunnel and flight testing. This adaptation was made by Bicknell (reference 5), by Silverstein and Katzoff (reference 6), and by Wright in some unpublished notes. One of the purposes of this paper is to compare the results obtained from each of these methods and to investigate the variation caused by the differences in assumptions and procedure.

The use of any of the momentum drag equations corrected for compressible flow involves a great many computations because a rather complicated expression must be calculated at a number of points in the wake. Considerable computation time may be eliminated by use of the integrating method outlined in reference 6. This method

evaluates the proportionality factor that exists between the drag and the average total head loss in the wake, a value which can be readily determined from graphical or numerical integration of point-by-point measurements, by use of an integrating manometer (reference 7), or by means of the averaging rake described in reference 6. Certain assumptions, principally that of a predetermined wake shape, are necessary for the evaluation of the proportionality factor. It is the purpose of this paper to determine experimentally the error entailed in the use of this method over a wide range of Mach numbers and wake shapes.

#### COMPARISON OF THE EQUATIONS OF JONES, BICKNELL, WRIGHT, AND SILVERSTEIN AND KATZOFF

The equations of Betz and Jones differ because of the original assumptions, but there is no significant difference between the results (reference 4). Jones' method has been used in the further developments because of the greater simplicity in computation procedure.

The defect in momentum caused by the drag of an airfoil is:

$$d_o = \rho \int_w \int v_2 (v_o - v_z) ds \quad (1)$$

assuming that the static pressures at stations 0 and 2 (fig. 1) are equal, which is true if station 2 is an infinite distance downstream. (All symbols used in this report are defined in figure 1 and in the appendix.) As the measurement of velocities at great distances from a body is impractical, it is necessary to convert equation (1) into one with terms that are measurable. In order to do this, Jones made the assumption that there is no mixing in the tubes of flow between plane 1, a measurement plane close behind the airfoil, and plane 2. This assumption permits the application of Bernoulli's equation between the two planes.

Then  $H_2 - p_2 = \frac{V_2^2}{2} = H_1 - p_0$  (2)

and equation (1) reduces to

$$c_{d_o} = \frac{2}{c} \int_w \frac{\sqrt{H_1 - p_1}}{\sqrt{H_0 - p_0}} \left( 1 - \frac{\sqrt{H_1 - p_0}}{\sqrt{H_0 - p_0}} \right) dy \quad (3)$$

or

$$c_{d_0} = \frac{2}{c} \int_w \frac{\sqrt{H_0 - p_1 - (H_0 - H_1)}}{\sqrt{H_0 - p_0}} \left( 1 - \frac{\sqrt{H_0 - p_0 - (H_0 - H_1)}}{\sqrt{H_0 - p_0}} \right) dy \quad (4)$$

As the Jones equation is applicable only to incompressible flow, the next refinement was to adapt it to compressible flow. Bicknell (reference 5) shows how Bernoulli's equation for compressible flow is applied to determine  $q$  and how the change in density caused by the static pressure difference between stations 0 and 1 may be taken into account by the assumption of adiabatic variations. With these modifications, Jones' equation becomes:

$$c_{d_0} = \frac{2}{c} \int_w \left[ \frac{\sqrt{\frac{H_1 - p_1 + [(p_1 - p_0) + \frac{p_1(p_1 - p_0)}{2\gamma(p_1)} + \dots]}{1 + \eta_1}}}{\sqrt{\frac{H_0 - p_0}{1 + \eta_0}}} \right] \sqrt{\frac{H_1 - p_1}{H_0 - p_0}} \frac{1 + \eta_0}{1 + \eta_1} dy \quad (5)$$

Bicknell did not consider the increase in temperature in the wake and the consequent density changes, an effect which is appreciable at high Mach numbers. This temperature rise was taken into account by Ray H. Wright in some unpublished work and by Silverstein and Katzoff in reference 6. The same assumptions are made in both developments, and the methods differ only in procedure.

On the assumption that the flow in each streamline tube in the wake between planes 1 and 2 is isentropic, the temperature rise in each lamina is equal to the energy difference between the work expended per unit time in overcoming the drag and the kinetic energy produced in the wake. The assumption of isentropic flow is analogous to the assumption made in the development of the incompressible flow equation that there is no mixing in the tubes of flow between planes 1 and 2 and, consequently, that the total head remains constant (reference 9). Experimental verification of the isentropic nature of the wake flow is given in reference 10, the results of which show that a constant stagnation temperature exists across the wake. Using the relations given in reference 6, the following equation can be developed for the profile drag coefficient. (See appendix.)

$$c_{d_0} = \frac{2}{c} \int_w \sqrt{\left(\frac{p_1}{p_0}\right)^{\frac{1}{\gamma}} \frac{(H_1-p_1)(1+\eta_0)}{(H_0-p_0)(1+\eta_1)}} \left[ \sqrt{\frac{1+0.202M^2 \left( \frac{H_1-p_0}{H_0-p_0} \right) \left( \frac{1+\eta_0}{1+\eta_2} \right)}{1+0.202M^2}} - \sqrt{\frac{(H_1-p_0)(1+\eta_0)}{(H_0-p_0)(1+\eta_2)}} \right] dy \quad (6)$$

Wright developed a similar equation, but introduced the factor  $G$  to define the relation  $\sqrt{\rho_2/\rho_0} = 1-G(1-q_2/q_0)$

where

$$G = \frac{1 - \sqrt{\frac{1+0.202M^2 q_2/q_0}{1+0.202M^2}}}{1 - q_2/q_0}$$

$$c_{d_0} = \frac{2}{c} \int_w \sqrt{\left(\frac{p_1}{p_0}\right)^{\frac{1}{\gamma}} \frac{(H_1-p_1)(1+\eta_0)}{(H_0-p_0)(1+\eta_1)}} \left[ 1-G \left( 1 - \frac{H_1-p_0}{H_0-p_0} \frac{1+\eta_0}{1+\eta_2} \right) - \sqrt{\frac{H_1-p_0}{H_0-p_0} \frac{1+\eta_0}{1+\eta_2}} \right] dy \quad (7)$$

$G$  is primarily a function of Mach number, and secondarily a function of  $(1-q_2/q_0)$ . The variation of  $G$  with these two factors is shown in figure 2.

Table I and figure 3 compare the profile drag coefficients as computed from point-by-point methods from wake measurements on a 66,2-420 airfoil (reference 7) throughout an extensive Mach number range. As expected, the Jones equation for incompressible flow gives the highest drag coefficient at any Mach number. Bicknell's equation is intermediate between compressible and incompressible flow because the temperature rise in the wake was neglected. The two corrected methods of computing the drag in compressible flow (equations (6) and (7)) give results which differ only because of inaccuracies in plotting and computing.

A comparison of the drag coefficients at a high Mach

number gives an indication of the magnitudes of the various corrections. At  $M = 0.6$ , the percentage difference between Jones' and Bicknell's equations is 7 percent, indicating the effect of compressibility, the inclusion of the  $1+\eta$  factor, and the density changes due to static pressure differences. The effect of the temperature rise in the wake on the density causes a further correction of 6 percent, as shown by the percentage difference between Bicknell's and the Silverstein-Katzoff or Wright equation. The total value of the corrections to the Jones equation at this Mach number is 13 percent. Assuming that equations (6) and (7) give fully corrected results, the error from the Jones equation will exceed 2 percent if it is used above a Mach number of 0.1, and the error from Bicknell's equation will exceed 2 percent if used above a Mach number of 0.25. The error in the Jones equation increases roughly as  $M^{1.3}$ ; that of the Bicknell equation increases approximately in proportion to  $M$ .

It should be noted that the drag results determined in NACA high-speed wind tunnels by the momentum method have included all of the compressibility and temperature corrections outlined above.

#### INTEGRATING METHODS

Equations (6) and (7) are somewhat unsatisfactory for wind-tunnel or flight test work because of the lengthy computations involved; the equations must be evaluated for a number of points throughout the wake, and the results graphically or numerically integrated. As developed in reference 6, a simplification results from the fact that the profile drag coefficient is roughly proportional to the average total-head loss in the wake.

$$c_{d_0} = \frac{F}{c} \int_w \frac{H_0 - H_1}{H_0 - p_0} dy \quad (8a)$$

or

$$c_{d_0} = F_i \frac{w}{c} \frac{(H_0 - H_1)_{av}}{H_0 - p_0} \quad (\text{incompressible flow}) \quad (8b)$$

$$c_{d_0} = F_c \frac{w}{c} \frac{(H_0 - H_1)_{av}}{H_0 - p_0} \quad (\text{compressible flow}) \quad (8c)$$

The value of  $(H_0 - H_1)_{av}$  may be determined from a graph-

ical or numerical average of point-by-point measurements, directly by use of an integrating manometer (reference 8), or by the averaging rake described in reference 6.  $F_i$  is a proportionality factor for incompressible flow and is a function of  $\frac{P_1 - P_0}{H_0 - P_0}$ ,  $\frac{(H_0 - H_1)_{\max}}{H_0 - P_0}$ , and wake shape. It is

evaluated in reference 6 on the assumption that the wake conforms to a cosine-squared curve. Equations (3) and (8b) are evaluated in terms of this assumption and equated; then  $F_i$  is determined for a range of values of  $P_1 - P_0 / H_0 - P_0$  and  $(H_0 - H_1)_{\max} / H_0 - P_0$ . The variation of  $F_i$  with these two factors in the range normally encountered is shown in figure 4.

To correct the integrating method for compressibility and temperature effects, Silverstein and Katzoff (reference 6) determined  $F_c$ , the proportionality factor in compressible flow, by evaluating equations (6) and (8c) in terms of the assumption of a cosine-squared wake form, equating and solving for  $F_c$ . Values of  $F_c/F_i$  so determined are presented in table I of reference 6 as a function of  $\frac{(H_0 - H_1)_{\max}}{H_0 - P_0}$ ,  $\frac{P_1 - P_0}{H_0 - P_0}$ , and Mach number.

In using this method in wind-tunnel testing, it has been found convenient to determine  $F_i$  first and then modify it by the ratio  $F_c/F_i$  for the Mach number of the test velocity. Thus,

$$c_{d_0} = F_c \frac{w}{c} \frac{(H_0 - H_1)_{av}}{H_0 - P_0} \quad (8c)$$

$$= \frac{F_c}{F_i} \left[ F_i \frac{w}{c} \frac{(H_0 - H_1)_{av}}{H_0 - P_0} \right] \quad (9)$$

The variation of  $F_c/F_i$  with the three variables involved is given in figure 5.

It is evident from equation (9) that any error involved in the assumption of a cosine-squared wake form is primarily a function of  $F_i$  and secondarily a function of  $F_c/F_i$ . In order to eliminate any error in  $F_i$ , the following equation may be used since  $F_c/F_i$  shows the

relation between compressible and incompressible flow:

$$c_{d_0} = \frac{F_c}{F_i} \left[ \begin{array}{l} \text{Drag as determined by the} \\ \text{Jones equation (equation (3))} \end{array} \right] \quad (10)$$

The accuracy of the integrating method was investigated from the wake measurements obtained in reference 7. The profile drag coefficients were determined by point-by-point computations and the integrating method from the wake surveys which were made through a Mach number range of 0.188 to 0.658. Table II compared the results and shows that the coefficients determined by equation (9) are the same as those obtained in equation (10). In other words,  $F_i$  contains very little error for the wake form encountered in the tests. Comparison of the results of equations (10) and (6) show that the error in  $F_c/F_i$  is also negligible. At all speeds below the critical, the greatest difference between the integrating method and the point-by-point computation of equation (6) was 1.7 percent.

Wakes may vary considerably in shape; so several different forms were assumed, to determine the effect of extreme departures from the cosine-squared assumption. The profile drag coefficient for an unsymmetrical, a triangular, a rectangular, and a true cosine-squared wake form (fig. 6) were computed by the point-by-point methods and by the integrating method using  $F_i$  and  $F_c/F_i$ . The unsymmetrical wake is the type found behind a wing-nacelle junction and is due to the combination of the wakes of each. Table III shows that the actual shape has little effect upon the error in the profile drag coefficient as computed from the cosine-squared assumption. The percentage errors between the integrating and the point-by-point computation methods show the error in  $F_i$ ,  $F_c/F_i$ , and the product of the two. There appears to be no consistency as the wake form departs more and more from the cosine-squared shape; the major portion of the error is probably due to inaccuracies in plotting and computing.

#### CONCLUSIONS

1. The Jones equation has less than a 2-percent error due to compressibility effects, up to a Mach number of 0.1. This error increases rapidly with Mach number, roughly in proportion to  $M^{1.3}$ .

2. The Bicknell equation has less than a 2-percent error up to a Mach number of 0.25 due to compressibility effects. This error increases approximately in proportion to Mach number.

3. The integrating method of Silverstein and Katzoff gives results which agree within 2 percent with the results of the integration of point-by-point computations (corrected for compressibility effects) at least up to the critical Mach number for normal wing wakes. The additional error due to extreme variation in wake shape is less than 1.5 percent.

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#### APPENDIX

Definitions of symbols used in this report:

- q dynamic pressure =  $1/2\rho V^2 = H-p$
- p static pressure
- H total pressure
- V velocity
- $\rho$  density
- T absolute temperature
- dS elemental area
- y vertical distance from wake center
- w width of wake
- c airfoil chord
- $\gamma$  ratio of specific heat at constant pressure to the specific heat at constant volume

M Mach number, the ratio of the stream velocity to the local velocity of sound

F proportionality factor. The subscript i denotes incompressible flow and c, compressible flow

$c_{d_0}$  section profile drag coefficient

$1+\eta$  a compressibility correction factor

$$q = \rho \frac{V^2}{2} = \frac{H-p}{1+\eta} q_i \text{ in compressible flow}$$

where  $1+\eta = 1 + \frac{M^2}{4} + \frac{M^4}{40} + \dots$

The subscripts o, i, and s denote the location of the measured or computed values: o denotes the free stream; i denotes the measurement plane, close behind the airfoil; s denotes the value at the plane located an infinite distance downstream from the airfoil,

The derivation of equation (6), according to the relations set up in reference 6, is as follows:

$$d = \int_w \rho_s V_s (V_o - V_s) dy_s \quad (a)$$

$$c_{d_0} = \frac{2}{c} \int_w \frac{\rho_s V_s}{\rho_o V_o} \left( 1 - \frac{V_s}{V_o} \right) dy_s \quad (b)$$

For continuity:  $\rho_i V_i dy_i = \rho_s V_s dy_s$

$$c_{d_0} = \frac{2}{c} \int_w \frac{\rho_i V_i}{\rho_o V_o} \left( 1 - \frac{V_s}{V_o} \right) dy_i \quad (c)$$

The difference between the work expended per unit time and the kinetic energy produced in the wake is:

$$\rho_s V_s V_o (V_o - V_s) dy_s - \frac{1}{2} \rho_s V_s (V_o - V_s)^2 dy_s = \frac{1}{2} \rho_s V_s (V_o^2 - V_s^2) dy_s$$

If this difference remains in the lamina as heat, the temperature rise is:

$$T_2 - T_1 = \frac{\frac{1}{2} p_2 V_2 (V_{c_0}^2 - V_2^2) dy_2}{c_p p_2 V_2 dy_2} \quad \dots (d)$$

where

$T$  absolute temperature, deg. C

$c_p$  specific heat at constant pressure

Substitution of the numerical value of  $c_p$  and division by  $T_0 = 0.000231 V_{c_0}^2$  leads to

$$\frac{T_2 - T_0}{T_0} = 0.202 \frac{V_2^2}{V_{c_0}^2} \left( 1 - \frac{V_2^2}{V_0^2} \right) \quad \dots (e)$$

$$\text{Since } p_0 = p_2, \frac{p_0}{p_2} = \frac{T_2}{T_0} = 1 + 0.202 \frac{V_2^2}{V_{c_0}^2} \left( 1 - \frac{V_2^2}{V_0^2} \right) \quad \dots (f)$$

$$\frac{V_2}{V_0} = \sqrt{\frac{p_0}{p_2} \frac{(H_2 - p_2)(1 + \eta_0)}{(H_0 - p_0)(1 + \eta_2)}} = \sqrt{\frac{p_0}{p_2} \frac{(H_1 - p_0)(1 + \eta_0)}{(H_0 - p_0)(1 + \eta_2)}} \quad \dots (g)$$

Solving these two equations simultaneously for  $p_0/p_2$

$$\frac{p_0}{p_2} = \frac{1 + 0.202 M^2}{1 + 0.202 M^2 \left( \frac{H_1 - p_0}{H_0 - p_0} \right) \left( \frac{1 + \eta_0}{1 + \eta_2} \right)} \quad \dots (h)$$

where  $M = V_0/V_{c_0}$  and is the Mach number.

$$\frac{V_1}{V_0} = \sqrt{\frac{p_0}{p_1} \frac{(H_1 - p_1)(1 + \eta_0)}{(H_0 - p_0)(1 + \eta_1)}} \quad \text{and} \quad \frac{p_1 V_1}{p_0 V_0} = \sqrt{\frac{p_1}{p_0} \frac{(H_1 - p_1)(1 + \eta_0)}{(H_0 - p_0)(1 + \eta_1)}} \quad \dots (i)$$

$$\frac{p_1}{p_0} = \frac{p_1}{p_2} \times \frac{p_2}{p_0} \quad \dots (j)$$

Since the flow between planes 1 and 2 is isentropic:

$$\frac{p_1}{p_2} = \left( \frac{p_1}{p_0} \right)^{\frac{1}{\gamma}} = \left( \frac{p_1}{p_0} \right)^{\frac{1}{\gamma}} \quad (k)$$

Substituting equations (k) and (f) in equation (j); and (j) into (i):

$$\frac{p_1 V_1}{p_0 V_0} = \sqrt{\left( \frac{p_1}{p_0} \right)^{\frac{1}{\gamma}} \frac{(H_1 - p_1)(1 + \eta_0)}{(H_0 - p_0)(1 + \eta_1)}} \times \sqrt{\frac{1 + 0.202M^2 \left( \frac{H_1 - p_0}{H_0 - p_0} \right) \left( \frac{1 + \eta_0}{1 + \eta_2} \right)}{1 + 0.202M^2}}$$

$$c_{d_0} = \frac{2}{c} \int_w \sqrt{\left( \frac{p_1}{p_0} \right)^{\frac{1}{\gamma}} \frac{(H_1 - p_1)(1 + \eta_0)}{(H_0 - p_0)(1 + \eta_1)}} \left[ \sqrt{\frac{1 + 0.202M^2 \left( \frac{H_1 - p_0}{H_0 - p_0} \right) \left( \frac{1 + \eta_0}{1 + \eta_2} \right)}{1 + 0.202M^2}} - \sqrt{\frac{(H_1 - p_0)(1 + \eta_0)}{(H_0 - p_0)(1 + \eta_2)}} \right] dy \quad (6)$$

Wright used the relation  $\sqrt{p_2/p_0} = 1 - G(1 - q_2/q_0)$

where

$$G = \frac{1 - \sqrt{p_2/p_0}}{1 - q_2/q_0} = \frac{1 - \sqrt{\frac{1 + 0.202M^2 q_2/q_0}{1 + 0.202M^2}}}{1 - q_2/q_0}$$

to obtain equation (7).

$$c_{d_0} = \frac{2}{c} \int_w \sqrt{\left( \frac{p_1}{p_0} \right)^{\frac{1}{\gamma}} \frac{(H_1 - p_1)(1 + \eta_0)}{(H_0 - p_0)(1 + \eta_1)}} \left[ 1 - G \left( 1 - \frac{H_1 - p_0}{H_0 - p_0} \frac{1 + \eta_0}{1 + \eta_2} \right) - \sqrt{\frac{H_1 - p_0}{H_0 - p_0} \frac{1 + \eta_0}{1 + \eta_2}} \right] dy$$

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TABLE I

$c_d$  vs  $M$  as computed by the various momentum drag methods from tests on a 66,2-420 airfoil

| Mach number | Jones' equation (3) | Bicknell's equation (5) | Silverstein-Katzoff equation (6) | Wright's equation (7) |
|-------------|---------------------|-------------------------|----------------------------------|-----------------------|
| .188        | .00568              | .00564                  | .00556                           | .00556                |
| .271        | .00637              | .00628                  | .00616                           | .00618                |
| .328        | .00629              | .00622                  | .00612                           | .00613                |
| .383        | .00637              | .00623                  | .00607                           | .00603                |
| .515        | .00698              | .00659                  | .00633                           | .00637                |
| .594        | .00756              | .00703                  | .00662                           | .00656                |
| .638        | .00888              | .00838                  | .00773                           | .00771                |
| .658        | .01218              | .01152                  | .01066                           | .01031                |

TABLE II

$c_{d_0}$  vs  $M$  to determine the error in  $F_i$  and  $F_c/F_i$

| Mach number | Integrating method $\times F_i \times F_c/F_i$ equation (9) | Jones' equation $\times F_c/F_i$ equation (10) | Silverstein-Katzoff equation (6) |
|-------------|---|--|----------------------------------|
| 0.188       | 0.00559   | 0.00560  | 0.00556                          |
| .271        | .00621  | .00618   | .00616                           |
| .328        | .00604  | .00602   | .00612                           |
| .383        | .00599  | .00600   | .00607                           |
| .515        | .00632  | .00629   | .00633                           |
| .594        | .00662  | .00659   | .00662                           |
| .638        | .00759  | .00759   | .00773                           |
| .658        | .01010  | .01034   | .01066                           |

| TABLE III<br>$c_{d_0}$ vs wake form to determine the error in $F_i$ and $F_c/F_i$ |                                  |  |               |  |               |
|---|----------------------------------|--|---------------|--|---------------|
| Wake form   | Silverstein-Katzoff equation (6) | Jones' equation $\times F_c/F_i$ equation (10) | Percent error | Integrating method $\times F_c/F_i$ equation (9) | Percent error |
| $\cos^2 \theta$   | 0.00898                          | 0.00899  | +0.1          | 0.00904  | +0.7          |
| Triangular  | .00912                           | .00902   | -1.1          | .00904   | -.9           |
| Wing-nacelle  | .01314                           | .01295   | -1.4          | .1308  | -,5           |
| Rectangular   | .01786                           | .01773   | -.7           | .01807   | +1.2          |

Incompressible flow

$$p_0, H_0, T_0, \rho_0, V_0$$

$$q_0 = H_0 - p_0$$

$$H_1, p_1, V_1$$

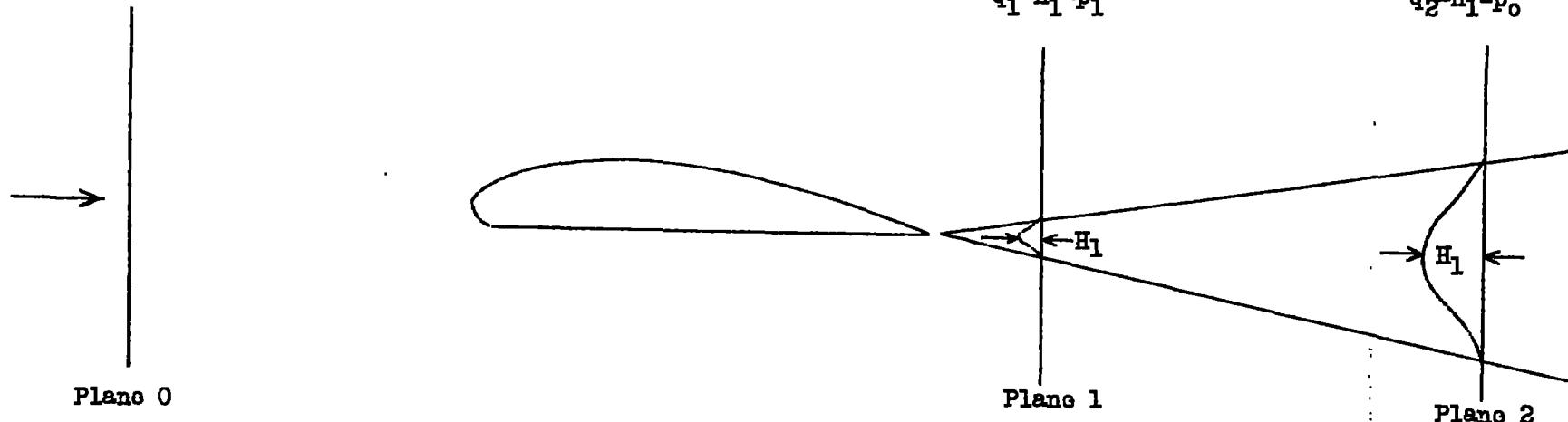
$$T_0, \rho_0$$

$$q_1 = H_1 - p_1$$

$$H_1, p_0, V_2$$

$$T_0, \rho_0$$

$$q_2 = H_1 - p_0$$



Compressible flow

$$p_0, H_0, T_0, \rho_0, V_0$$

$$q_0 = \frac{H_0 - p_0}{1 + \eta_0}$$

$\gamma$  = Vertical distance  
from wake center

$$p_1, V_1$$

$$T_1, \rho_1$$

$$q_1 = \frac{H_1 - p_1}{1 + \eta_1}$$

$$p_0, V_2$$

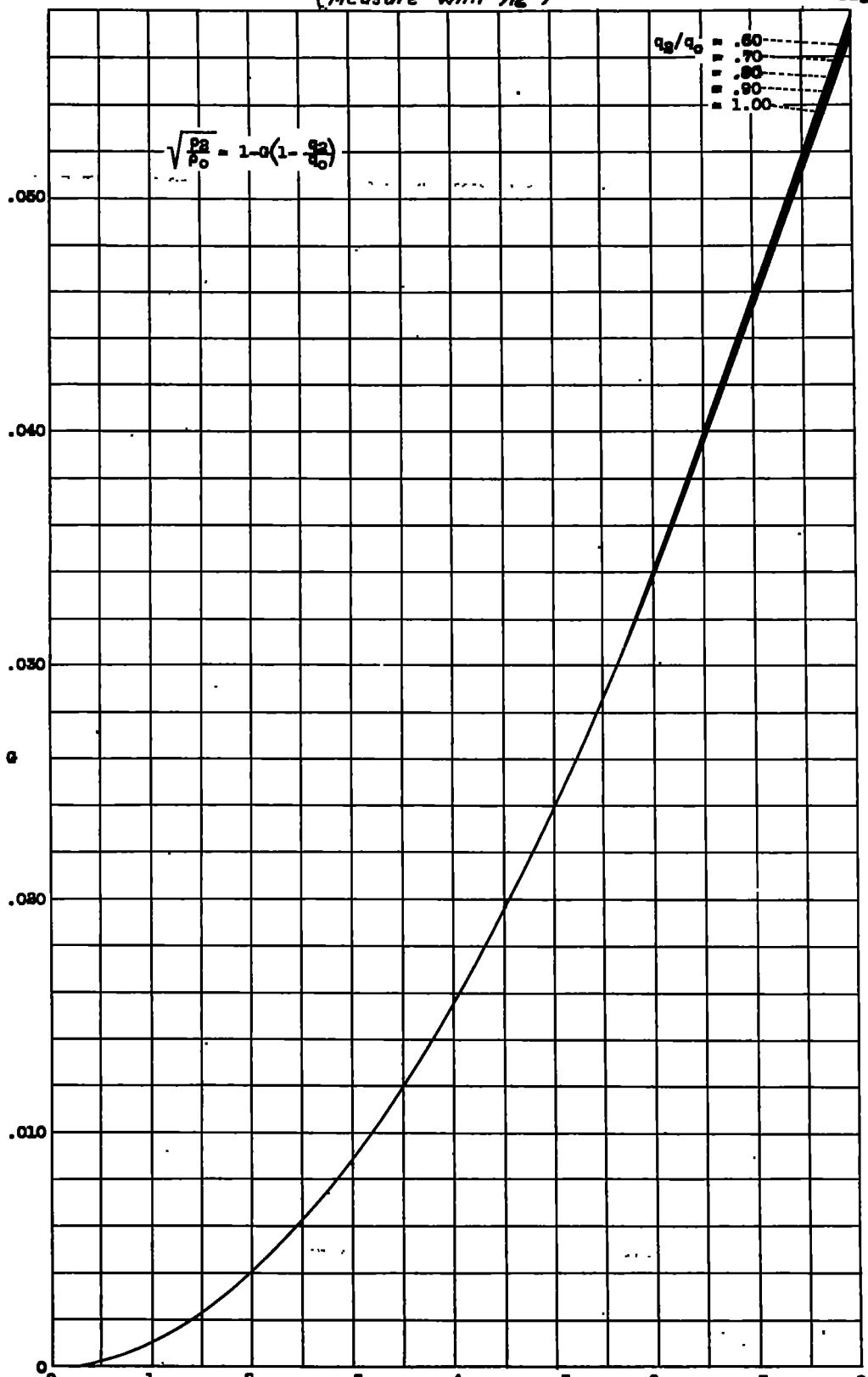
$$T_2, \rho_2$$

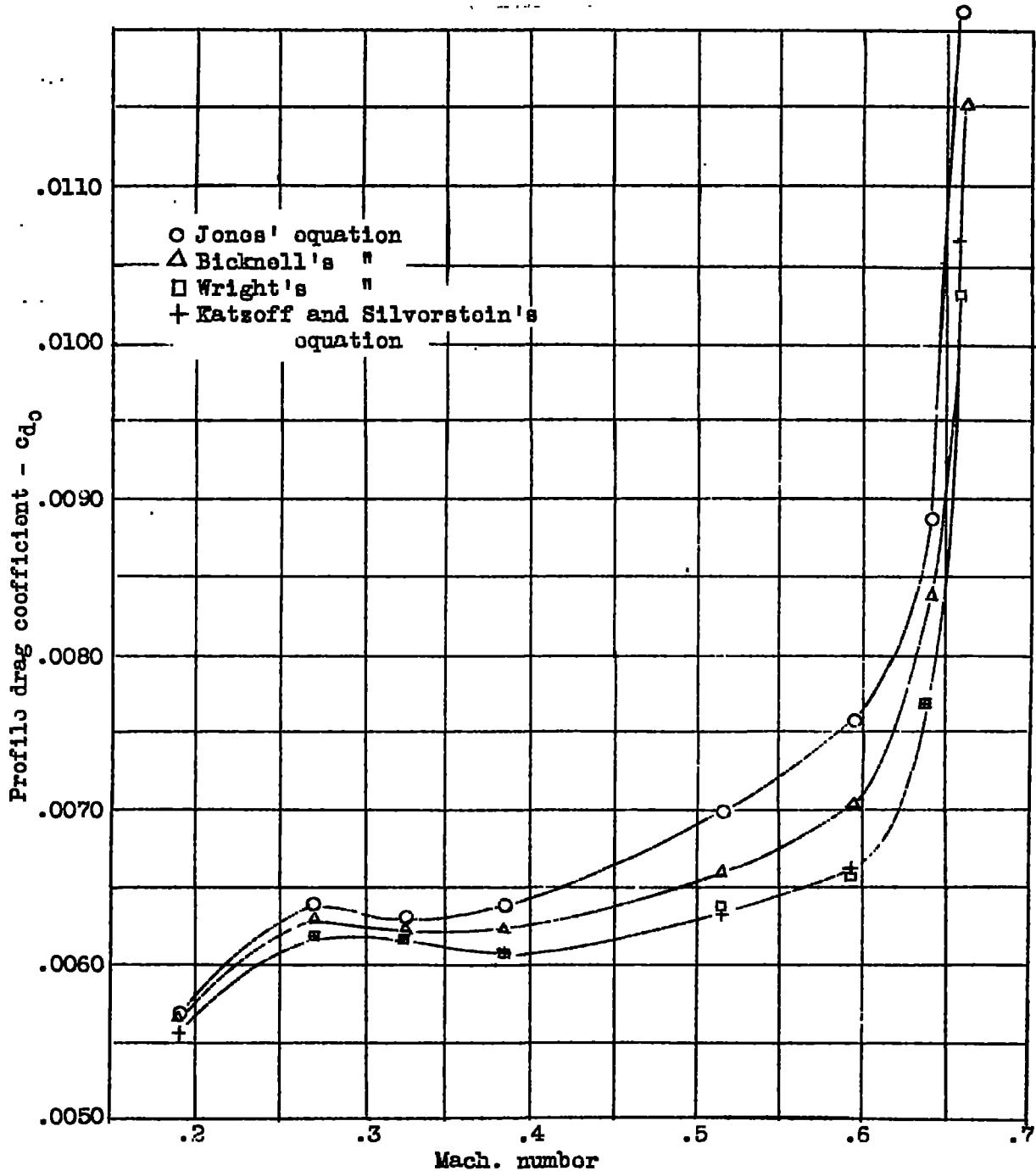
$$q_2 = \frac{H_1 - p_0}{1 + \eta_2}$$

Figure 1.- Definition of terms used in the momentum equations.

(Measure with  $\frac{5}{16}$ "")

Fig. 2

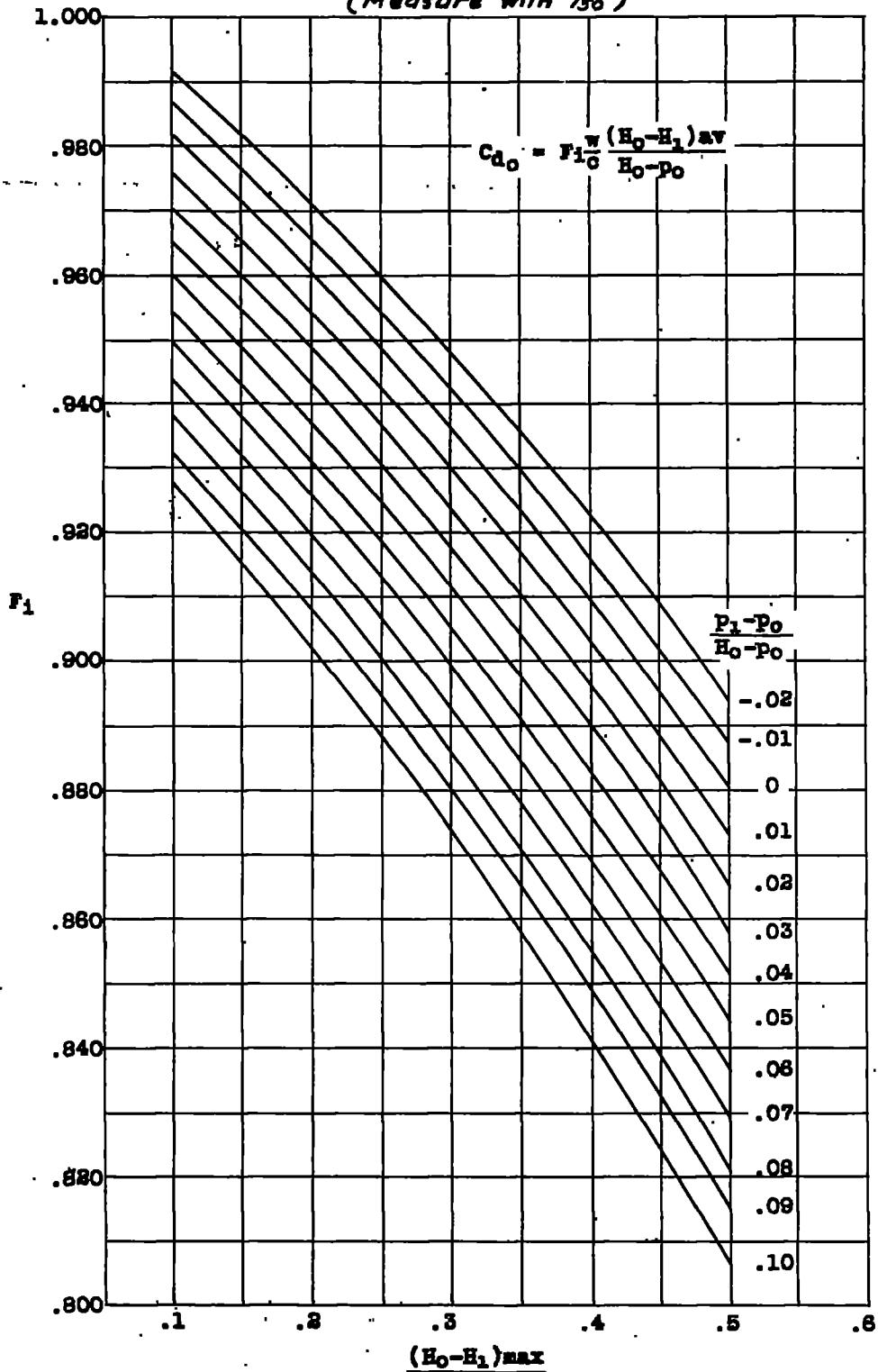
Figure 2.-  $g$  vs Mach number

Figure 3.-  $c_{d_0}$  vs. Mach number by the various point-by-point methods.

NACA

(Measure with  $\frac{1}{50}$ "')

Fig. 4

Figure 4.-  $F_1$  vs  $\frac{(H_0 - H_1)_{\text{max}}}{H_0 - P_0}$  for constant values of  $\frac{P_1 - P_0}{H_0 - P_0}$ .

NACA

(Measure with  $\gamma_0$ )

Fig. 5

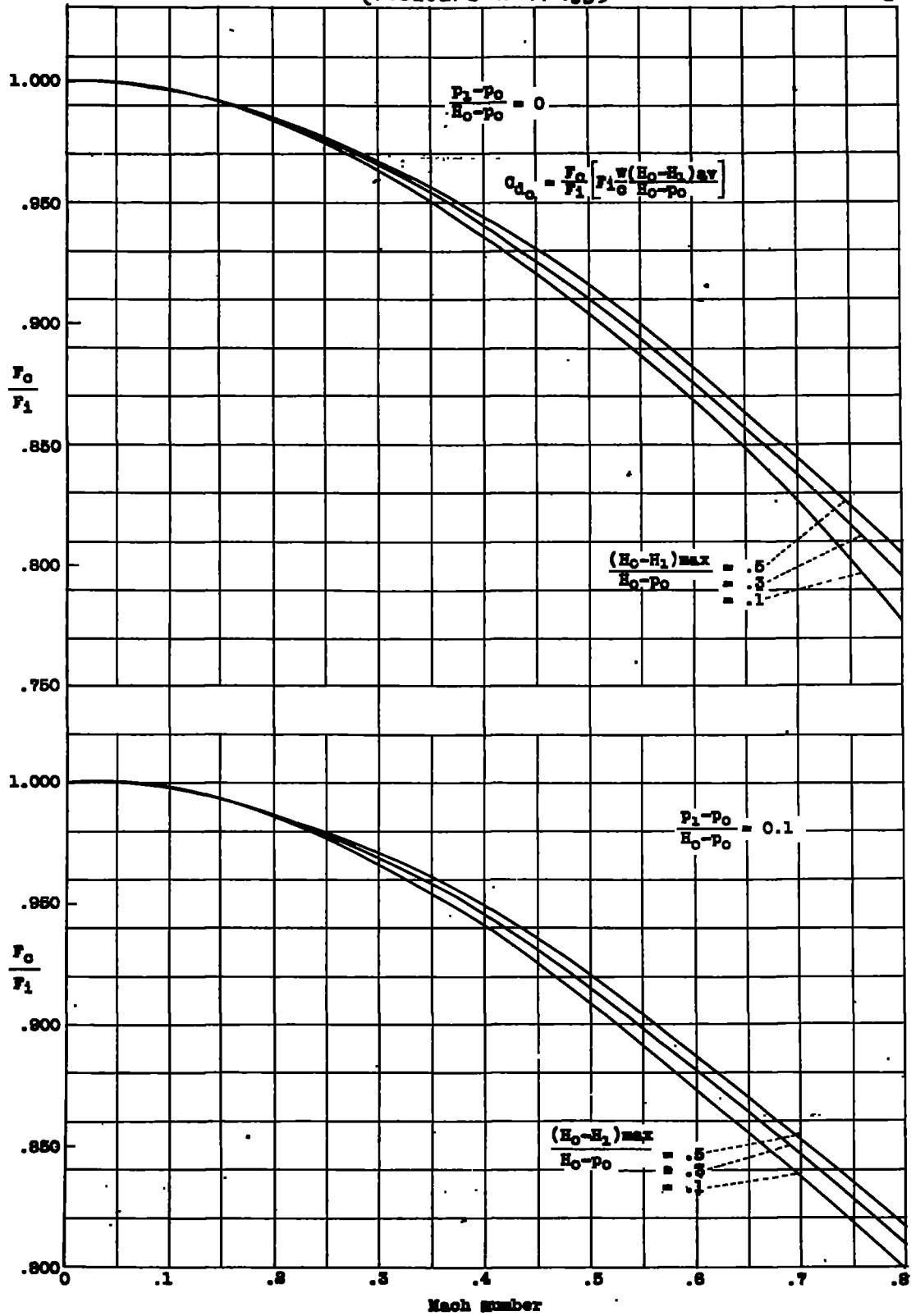


Figure 5.-  $P_0/P_1$  vs Mach number for constant values of  $P_1 - P_0/H_0 - P_0$  and  $(H_0 - H_1)_{\max}/H_0 - P_0$ .

NACA

Fig. 6

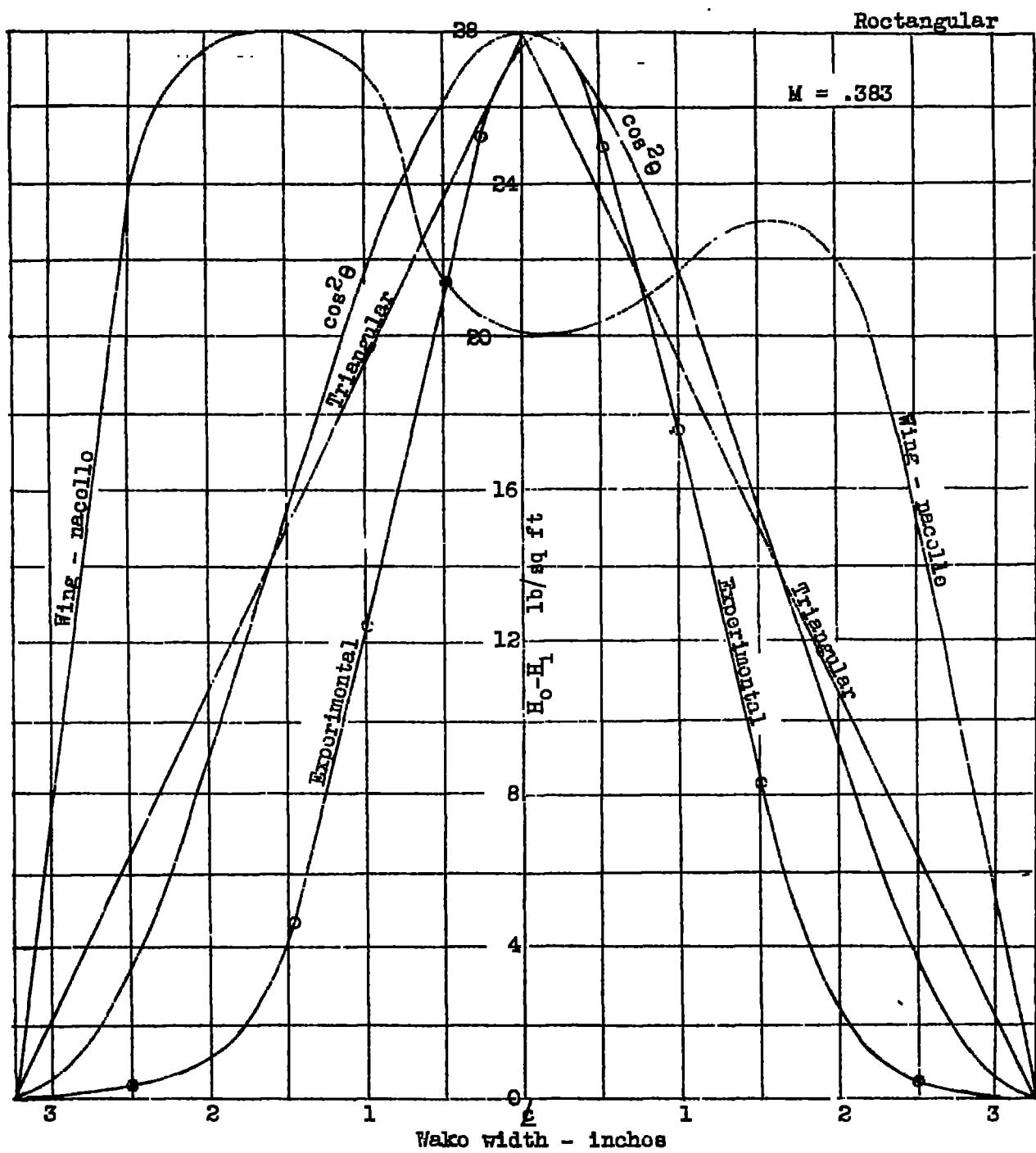


Figure 6.- Wake forms.